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Unusual diffusion in a quasi-two-dimensional granular gas

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We have studied diffusion in a quasi-two-dimensional granular gas composed of plastic balls confined in a vertically vibrating thin box. The horizontal motion of the balls in the box is found to follow the Langevin equation with the top and bottom of the box acting on the balls with a viscous drag like that in a fluid. Surprisingly, we find that both the granular temperature and the diffusion constant increase with the number of balls (N) in the box for small N. The unusual diffusion can be explained by a two-state model, in which a ball is in contact with two effective temperature baths due to collisions with the top or bottom of the box and collisions with other balls.

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From simple kinetic theory, the diffusion rate of a molecule in an ordinary gas decreases with density [1]. The reason is simple: higher density implies shorter mean free path, and hence, more frequent collisions that randomize molecular motion. If the average kinetic energy of a molecule increases after collision, it is possible to observe an unusual diffusion phenomenon such that the diffusion rate increases with density. In a granular gas, collisions are dissipative and the total kinetic energy of the particles involved in a collision decreases after the collision. Hence, one would not expect to find unusual diffusion in a granular gas. Here, we report a quasi-two-dimensional granular gas that exhibits this unusual diffusion, which can be explained using a two-state model based on the energy flow within the gas. We believe that the explanation of the unusual diffusion by this model provides a way to tackle general nonequilibrium steady-state problems, which involve several stochastic processes.

A granular gas, which consists of macroscopic particles colliding with each other dissipatively, can be sustained in a nonequilibrium steady state when the energy dissipated in inelastic collision is balanced by external energy sources [2]. However, the behavior of the granular gas depends on the details of energy injection [3]. If energy is injected through boundaries, unavoidable spatial inhomogeneity will complicate our understanding of the granular gas [4–8]. Nevertheless, it is possible to achieve uniform heating and cooling if the particles are confined in a vertically vibrating thin box [9–11]. A particle in the box may gain or lose its horizontal speed when colliding with the top or the bottom or with other particles. On average, energy is transferred from the vertical motion of a particle to its horizontal motion and dissipated by inelastic collisions [11–13]. One may consider the horizontal granular gas to be in contact with two effective temperature baths: a high-temperature bath, in which a particle gains its horizontal speed on average, and a low-temperature bath, in which a particle loses its horizontal speed on average. Based on this model, physical properties such as granular temperature, diffusion constant and velocity distribution of such quasi-two-dimensional granular gases (Q2DGSs) can be calculated. We find that the diffusion constant of the Q2DGS *increases* in the number of balls in the box in the dilute regime.

A schematic diagram of our experimental setup is shown in Fig. 1. Plastic balls in four different colors, each of them having mass $m=0.115$ g and diameter $d=5.94$ mm, are put in a circular acrylic box of internal height $h=12.5$ mm and internal radius $R=150$ mm. The internal walls of the box are coated with antistatic spray to avoid electrostatic effects. The box is mounted on a vibration platform which oscillates up and down sinusoidally at frequency $F=20$ Hz and amplitude $A=1.84$ mm. A fast color camera (C), of 1024×1024 pixel^2 with 10 μm spatial resolution, is used to take images from above at 1000 fps. Using balls of different colors together with a fast color camera, we can track the horizontal trajectories of the balls even at high densities. From the trajectories, we obtain the horizontal velocity components and compute the velocity distributions. We also calculate the mean-square displacement (MSD) in the horizontal plane.

Figure 2 shows that $\text{MSD}=\langle v_T^2\rangle t^2$ for $t<0.01$ s. This implies that the balls move in a straight line with a mean square speed $\langle v_T^2\rangle$ at small t. We define the granular temperature as $T_g=\frac{1}{2}m\langle v_T^2\rangle$ which is the average horizontal kinetic energy of a ball. On the other hand, MSD increases linearly with t for $t>0.1$ s, indicating that the balls move diffusively with a diffusion constant D given by $\text{MSD}=4Dt$. The crossover from ballistic motion to diffusive motion takes place at $t\approx0.025$ s, which is half the shaking period. This is reason-

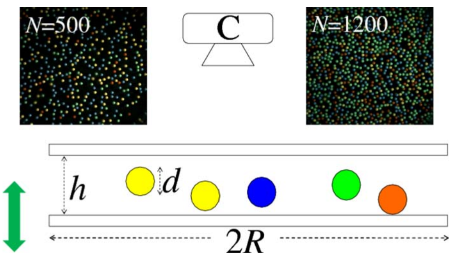

FIG. 1. (Color online) The schematic diagram of the experimental apparatus. The two images shown in the figure were taken at $N=500$ (left) and 1200 (right).

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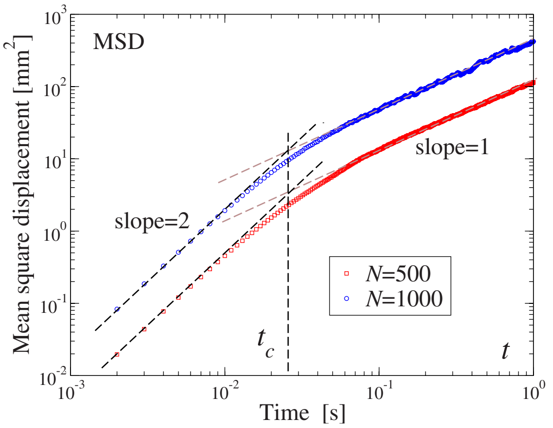

FIG. 2. (Color online) Mean square displacement for $N=500$ (red squares) and 1000 (blue circles).

able because if a ball moves up and down with the shaking frequency, the time between collisions with the top/bottom is roughly half the shaking period. Interestingly, MSD for $N=500$ is smaller than that for $N=1000$, implying faster diffusion at higher density. When we repeat the experiment for N between 50 and 1800, we find that both D and T_g increase with N for $N<N_c\approx 900$ as shown in Fig. 3. For $N>N_c$, D decreases with N while T_g stays nearly constant and then decreases when $N>1500$. This unusual diffusion can also be observed in experiments using copper spheres and in molecular dynamic simulations.

To understand the results of our experiments, we start from the observation of the motion of a ball. Its vertical motion, which is not detected by our camera, can be rather complicated [13,14]. If the ball bounces up and down between the top and the bottom with the oscillation frequency of the box, its vertical speed should be of the order of $v_z\approx 2(h-d)\ F=260$ mm/s. Clearly, a ball in the granular gas experiences two types of collision: collision with the top or the bottom (type-1) and collision with another ball (type-2). For a ball with nearly zero horizontal speed, a type-1 collision gives the ball a small random horizontal speed $v_1(\ll v_z)$ due to the imperfect spherical shape of the ball and the surface roughness of the top and bottom. For a fast moving ball, its horizontal speed is reduced after a type-1 collision, due to the fact that the tangential restitution coefficient is less than one. Hence, on a time scale longer than the inverse collision rate, the type-1 collisions act on the balls with a viscous drag like that in a viscous fluid. On the other hand, a slowly moving ball will most likely acquire a large horizontal speed comparable to v_z after a type-2 collision. Since the velocities of the colliding balls change according to their three-dimensional kinematics, the impulse to a ball due to a type-2 collision can be treated as stochastic when projected onto the horizontal plane [12–14]. Therefore, the Q2DGS is analogous to a colloidal system without hydrodynamic interaction and the motion of the balls should follow the Langevin equation: $m\frac{dv}{dt}=-\gamma v+\xi$, where γ is the effective viscosity and ξ is the stochastic force due to type-1 and type-2 collisions. The solution to the Langevin equation implies that MSD grows linearly with time at long time and quadratically at short time [1]. This explains why the balls move ballistically at short time and diffusively at long time.

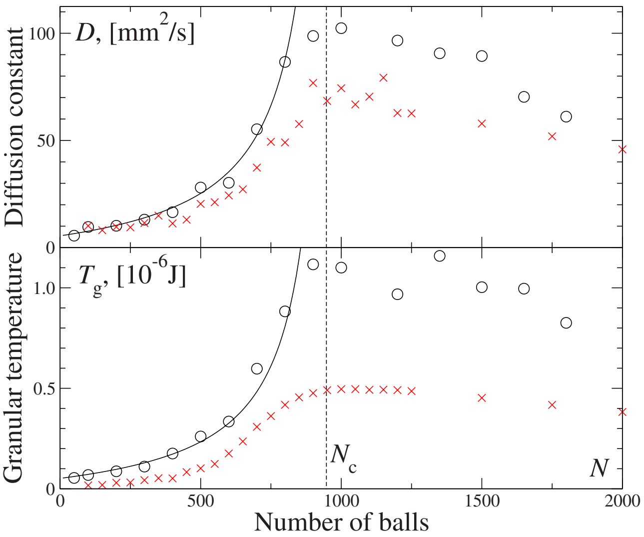

FIG. 3. (Color online) Variation of the diffusion constant D and the granular temperature T_g in the granular gas at different N. The lines in the graphs are the theoretical predictions from the two-state model and the crosses are the results from molecular dynamic simulations.

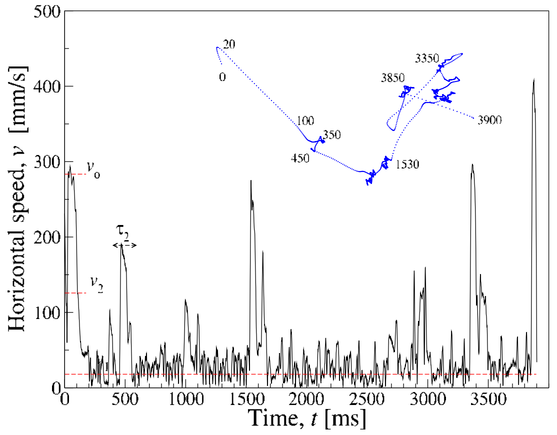

FIG. 4. (Color online) Time record of the horizontal speed of a ball at $N=500$. The inset shows the horizontal trajectory of the ball.

While the type-1 collision rate should be comparable to the box vibration frequency, the type-2 collision rate f should increase with N. To find the type-2 collision rate at different N, we need a more detailed examination of the motion of the balls. Figure 4 shows a typical trajectory of a ball together with the time record of its horizontal speed at $N=500$. In this trajectory the ball experiences type-2 collisions at $t=20,350,450,\ldots,$ ms. Consider the collision at $t=20$ ms. Right after this collision, the ball acquires a speed $v_0\approx 300$ mm/s, which is comparable to the vertical speed v_z and much higher than the random speed $(v_1\approx 25$ mm/s) due to type-1 collision. Then its speed drops to v_1 in $\tau_2\approx 100$ ms. From this observation, we can consider a ball in the Q2DGS to be in two possible states: a high-speed state (HSS), in which the ball moves nearly in a straight line with an average speed $\bar{v}_2$, and a low-speed state (LSS), in which the ball performs random motion with an average speed $\bar{v}_1$ due to type-1 collisions.

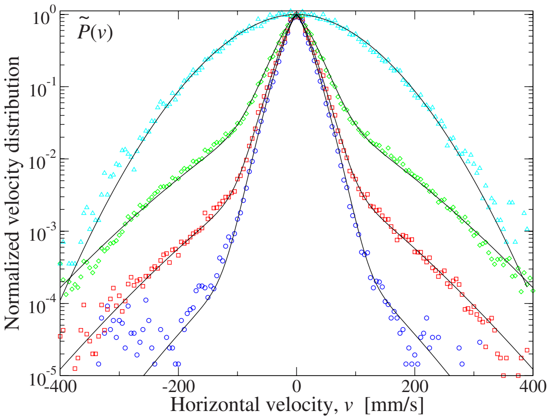

FIG. 5. (Color online) Normalized velocity distribution $\tilde{P}(v)$ for $N=50$ (blue circle), 200 (red square), 500 (green diamond), and 1800 (cyan triangle). The lines are obtained using expression (1).

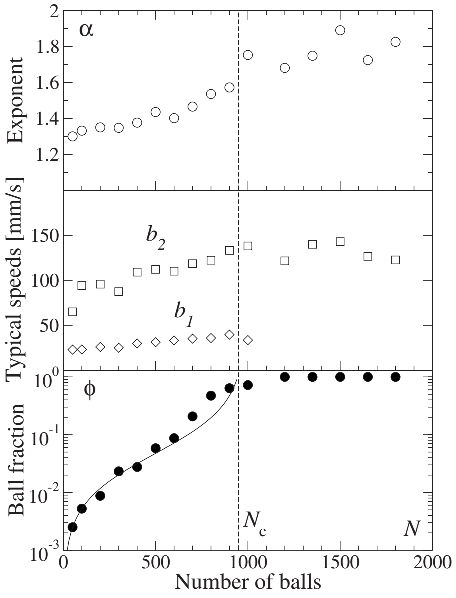

FIG. 6. Parameters: α (circle), b_1 (diamond), b_2 (square), and ϕ (filled circle) used for fitting the experimental normalized velocity distribution $\tilde{P}(v)$ to expression (1). The line in the ϕ vs N graph is obtained by expression (2).

Note that both HSS and LSS are nonequilibrium states because energy is not conserved in type-1 and type-2 collisions. Nevertheless, the velocity distribution of the balls in each of these two states should be a stretched exponential [3] of the form $e^{-|v/b|^{\alpha}}$ with $1<\alpha<2$. Hence, the velocity distribution $P(v)$ of the Q2DGS should be a sum of two stretched exponentials weighted by the ball fractions in each of the two states. Since the velocity distributions in the two orthogonal directions are identical, we use both components to construct the normalized velocity distribution $\tilde{P}(v)\equiv P(v)/P(0)$. It is found (see Fig. 5) that for $N\leq 1000$, $\tilde{P}(v)$ can indeed be fitted to

$$\tilde{P}(v) = (1-\phi)e^{-|v/b_1|^{\alpha}} + \phi e^{-|v/b_2|^{\alpha}}, \qquad (1)$$

with α increases with N. At higher densities, $\tilde{P}(v)$ can be fitted to a single stretched exponential $e^{-|v/b_2|^{\alpha}}$ with α varies between 1.6 and 1.9. Figure 6 shows the parameters obtained from the fitting. Since b_1 and b_2 are the typical speeds for the balls in HSS and LSS, they are of the same order of $\bar{v}_1$ and $\bar{v}_2$, respectively. The quantity ϕ is the fraction of balls in HSS and it is also the time fraction that a ball stays in HSS.

Let the lifetime of HSS be $\tau=\langle\tau_2\rangle$. It depends on the effective viscosity due to type-1 collisions. A ball in HSS sweeps an interaction area $2d\bar{v}_2\tau$ before it returns to LSS. The ball will stay in LSS for some time τ_1' until a type-2 collision excites it back to HSS. The mean squared distance moved during LSS is $4D_1\tau_1'$ and the area covered is $4\pi D_1\tau_1'$. Here, D_1 is the effective diffusion constant due to type-1 collisions and its value (≈ 5.5 mm^2/s) can be taken from the measurement at $N=50$. Since a ball experiences f type-2 collisions in one second, the fraction of time that a ball spends in HSS will be $\phi=f\tau$ and that for LSS will be $1-f\tau$. Therefore, the total interaction area covered in one second will be $a=f2d\bar{v}_2\tau+4\pi D_1(1-f\tau)$. The type-2 collision rate will be $\frac{pN}{\pi R^2}a$, where $\pi R^2=7.07\times 10^4$ mm^2 is the base area of the box and $p=d/h$ approximates the probability that two balls actually collide when their projected separation is less than d. Hence, we have $f=\frac{pN}{\pi R^2}[f2d\bar{v}_2\tau+4\pi D_1(1-f\tau)]$. After some algebra we arrive at

$$\phi = f\tau = \frac{2\pi D_1}{(\pi R^2)/(2pN\tau)-(d\bar{v}_2-2\pi D_1)}. \qquad (2)$$

When $f\tau=1$, most of the balls are in HSS. This happens when $N=N_c=945$ according to Eq. (2). Hence, the start of the flattening of T_g near N_c in Fig. 3 is a signature that the stochastic force ξ in the Langevin equation becomes dominated by that due to type-2 collisions. At higher densities, D decreases with N, as expected for an ordinary gas.

To see why D increases with N when $N<N_c$, we suppose that the Q2DGS is in contact with two effective temperature baths of temperature T_1 and T_2 due to type-1 and type-2 collisions, respectively. Clearly the coupling of the granular gas to T_1 and T_2 should be proportional to the fraction of time during which the ball is in random motion and in linear motion, respectively. So the granular temperature can be expressed as

$$T_g = (1-\phi)\kappa_1 T_1 + \phi\kappa_2 T_2, \qquad (3)$$

where κ_1 and κ_2 are the coupling constants to bath-1 and bath-2, respectively. Since there is net energy flow into the Q2DGS via type-2 collisions, there is always a temperature difference between bath-2 and the Q2DGS. Let $\bar{v}_0$ be the average speed of the ball right after a type-2 collision. Then the temperature of bath-2 should be $T_2=m\bar{v}_0^2/2$. Meanwhile, the mean speed of a ball in HSS is $\bar{v}_2=(\bar{v}_0+\bar{v}_1)/2\approx\bar{v}_0/2$ because $\bar{v}_0\gg\bar{v}_1$. So $\kappa_2=1/4$. When $N\approx N_c$ (i.e., $\phi\approx 1$), $T_g\approx T_2/4$. Our data shows that T_g at N_c is 1.1×10^{-6} J. Hence, $T_2=4.4\times 10^{-6}$ J. If type-2 collisions are rare, the Q2DGS can be considered to be in contact with bath-1 only. Since the energy of the Q2DGS is constant, the Q2DGS is in quasiequilibrium with bath-1 and hence $T_g = T_1$ and $\kappa_1 = 1$. The data at $N = 50$ (i.e., $\phi \approx 0$) shows that $T_1 = T_g = 0.054 \times 10^{-6}$ J.

Putting these values in expression (3) and using Eq. (2), we obtain a reasonable fit of the expected granular temperature from the two-state model for $N < N_c$, as shown in Fig. 3. By similar argument, the diffusion constant of the Q2DGS can be expressed as $D = (1-\phi)D_1 + \phi D_2$ with D_2 being the diffusion constant of a ball in HSS. At $N = N_c$, all the balls are in HSS and $D_2 = 100\ \mathrm{mm}^2/\mathrm{s}$ from our measurements. With this value for D_2, the diffusion constant from the two-state model fits the experimental data well for $N < N_c$, as shown in Fig. 3. For $N > N_c$, the number of events that a ball encounters another type-2 collision before returning to LSS become significant and the two-state model will not apply.

There has been experimental work on similar Q2DGS in which no unusual diffusion is reported. Baxter and Olafsen [9] used a two-layer setup to implement a Gaussian thermostat for their Q2DGS to obtain a Maxwell-Boltzmann velocity distribution. Reis *et al.* [11] constructed a Q2DGS very similar to ours except that the bottom of their vibrating box is rough. They studied the velocity distribution in terms of the Sonine polynomial expansion. Although they reported an increase in the granular temperature with density in a setup with smooth bottom, they did not study the diffusive behavior in this setup. In the two-layer or rough bottom systems, a ball in the Q2DGS could acquire substantial horizontal speed by colliding with either the first layer dimer or the rough bottom, respectively. Collision with another ball would not change its speed very much. There would be only one single state in these systems and the unusual diffusion would not be observed.

Clearly, the two-state model does not involve the collision details. As long as a particle can be excited to a high-speed state and stays there with a finite lifetime τ, the system will exhibit unusual diffusion when τ is less than the inverse excitation rate. This is indeed confirmed by our molecular dynamic simulations of the Q2DGS and the details will be given in another paper.

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